

# CP violation in the lepton flavor violating interactions $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$

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Physics Department, Middle East Technical University  
Ankara, Turkey

## Abstract

We calculate the possible CP violating asymmetries for LFV decays  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu\gamma$ . Our predictions depend on the chosen new model independent contribution. The result of measurements of such asymmetries for these decays will open a new window to determine the physics beyond the SM.

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\*E-mail address: eiltan@heraklit.physics.metu.edu.tr

# 1 Introduction

Lepton Flavor Violating (LFV) interactions are among the most promising candidates to understand the physics beyond the Standard model (SM). The improvement of their experimental measurements forces to make more elaborate theoretical calculations and to determine the unknown parameters existing in the models used.  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu\gamma$  decays are the examples for the LFV decays and the current limits for their branching ratios ( $BR$ ) are  $1.2 \times 10^{-11}$  [1] and  $1.1 \times 10^{-6}$  [2] respectively.

LFV interactions have been analysed in different models in the literature. They were studied in a model independent way in [3], in the framework of model III 2HDM [4], in supersymmetric models [5, 6, 7, 8, 9, 10, 11]. Recently, the electromagnetic suppression of the decay rate of  $\mu \rightarrow e\gamma$  has been predicted in [12]. Further, the processes  $\tau \rightarrow \mu\gamma$  and  $\mu \rightarrow e\gamma$  have been studied as probes of neutrino mass models in [13].

LFV processes do not exist in the Standard model (SM) if the Cabibbo-Kobayashi-Maskawa (CKM) type matrix in the leptonic sector vanishes and this stimulates one to go beyond. The general two Higgs doublet model, so called model III, permits such interactions which appear at least in the loop level, with the internal mediating neutral Higgs bosons  $h_0$  and  $A_0$ . Note that, in this case, there is no charged Flavor Changing (FC) interaction. There are large number of free parameters and their strength can be determined by the experimental data. Choice of complex Yukawa couplings causes the CP violating effects which can also provide comprehensive information in the determination of free parameters of the various theoretical models. Non-zero Electric Dipole Moments (EDM) of the elementary particles are sign of such violations.

In this work, we study the possibility of CP asymmetry  $A_{CP}$  of decays  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu\gamma$  in the model III. Even if the Yukawa couplings are taken as complex in general, the model III does not ensure CP asymmetry for the decays under consideration. However, by correcting the decay rates of these processes with the additional complex contribution, which may come from the physics beyond the model III, a measurable  $A_{CP}$  can be obtained. Here, we assume that the complexity of the new contribution is due to not the Yukawa type couplings, but probably to radiative corrections in this model. Therefore, the forthcoming experimental measurements of possible  $A_{CP}$  for both processes open a new window understand the physics beyond the SM.

The paper is organized as follows: In Section 2, we present the possible CP violating asymmetry for LFV decays  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu\gamma$ . Section 3 is devoted to discussion and our conclusions.

## 2 CP violation in LFV interactions $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$

Our starting point is type III 2HDM which permits flavor changing neutral currents (FCNC) at tree level. The Yukawa interaction for the leptonic sector in the model III is

$$\mathcal{L}_Y = \eta_{ij}^E \bar{l}_{iL} \phi_1 E_{jR} + \xi_{ij}^E \bar{l}_{iL} \phi_2 E_{jR} + h.c. , \quad (1)$$

where  $i, j$  are family indices of leptons,  $L$  and  $R$  denote chiral projections  $L(R) = 1/2(1 \mp \gamma_5)$ ,  $\phi_i$  for  $i = 1, 2$ , are the two scalar doublets,  $l_{iL}$  and  $E_{jR}$  are lepton doublets and singlets respectively. Here  $\phi_1$  and  $\phi_2$  are chosen as

$$\phi_1 = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix} \right] ; \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ H_1 + iH_2 \end{pmatrix} , \quad (2)$$

and the vacuum expectation values are

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} ; \langle \phi_2 \rangle = 0 . \quad (3)$$

With this choice, the SM particles can be collected in the first doublet and the new particles in the second one. Here the bosons  $H_1$  and  $H_2$  are the neutral CP even  $h^0$  and CP odd  $A^0$ , respectively since there is no mixing between CP even neutral Higgs bosons  $H^0$  and  $h^0$  at tree level. The part which produce FCNC (at tree level) is

$$\mathcal{L}_{Y,FC} = \xi_{ij}^E \bar{l}_{iL} \phi_2 E_{jR} + h.c. . \quad (4)$$

The Yukawa matrices  $\xi_{ij}^E$  have in general complex entries. Note that in the following we replace  $\xi^E$  with  $\xi_N^E$  where "N" denotes the word "neutral".

Now, let us consider the lepton number violating process  $\mu \rightarrow e\gamma$ . Here, we expect that the main contribution to this decay comes from the neutral Higgs bosons, namely,  $h_0$  and  $A_0$ , in the leptonic sector of the model III, (see Fig. 1). Using on-shell renormalization scheme the self energy diagrams vanish and only the vertex diagram (Fig. 1-c) contributes. By taking  $\tau$  lepton for the internal line, the decay width  $\Gamma$  becomes [14]

$$\Gamma(\mu \rightarrow e\gamma) = c_1(|A_1|^2 + |A_2|^2) , \quad (5)$$

where

$$\begin{aligned} A_1 &= Q_\tau \frac{1}{8 m_\mu m_\tau} \bar{\xi}_{N,\tau e}^E \bar{\xi}_{N,\tau\mu}^E (F_1(y_{h_0}) - F_1(y_{A_0})) , \\ A_2 &= Q_\tau \frac{1}{8 m_\mu m_\tau} \bar{\xi}_{N,e\tau}^{E*} \bar{\xi}_{N,\mu\tau}^{E*} (F_1(y_{h_0}) - F_1(y_{A_0})) , \end{aligned} \quad (6)$$

$c_1 = \frac{G_F^2 \alpha_{em} m_\mu^5}{32\pi^4}$  and the function  $F_1(w)$  reads

$$F_1(w) = \frac{w(3 - 4w + w^2 + 2\ln w)}{(-1 + w)^3}, \quad (7)$$

with  $y_H = \frac{m_\tau^2}{m_H^2}$ . In eq. (6),  $\bar{\xi}_{N,ij}^E$  is defined as  $\xi_{N,ij}^E = \sqrt{\frac{4G_F}{\sqrt{2}}} \bar{\xi}_{N,ij}^E$  and the amplitudes  $A_1$  and  $A_2$  have right and left chirality respectively. In our calculations we ignore the contributions coming from internal  $\mu$  and  $e$  leptons, respecting our assumption on the Yukawa couplings (see Discussion).

At this stage, we calculate the CP asymmetry  $A_{CP}$  of the process  $\mu \rightarrow e\gamma$ , given by the expression

$$A_{CP} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \quad (8)$$

where  $\bar{\Gamma}$  is the decay width for the CP conjugate process. However, in the framework of the model III,  $A_{CP}$  vanishes and one needs an extra quantity to switch on the CP asymmetry. Therefore, we assume that there is an additional small and complex contribution  $\chi$  due to the physics beyond the model III such that the factor  $\bar{\xi}_{N,\tau e}^E \bar{\xi}_{N,\tau\mu}^E (F_1(y_{h_0}) - F_1(y_{A_0}))$  is corrected as

$$\bar{\xi}_{N,\tau e}^E \bar{\xi}_{N,\tau\mu}^E (F_1(y_{h_0}) - F_1(y_{A_0})) + \chi.$$

Further, we force that the complexity of  $\chi$  comes from the possible radiative corrections but not from the Yukawa type couplings, in the model beyond the model III. This choice of  $\chi$  brings a non-zero CP violation for the process  $\mu \rightarrow e\gamma$ :

$$A_{CP} = 2 \frac{|\bar{\xi}_{N,\tau e}^E \bar{\xi}_{N,\tau\mu}^E| (F_1(y_{h_0}) - F_1(y_{A_0})) |\chi| \sin \theta_\chi \sin (\theta_{\tau e} + \theta_{\tau\mu})}{\Phi} \quad (9)$$

where

$$\begin{aligned} \Phi &= |\bar{\xi}_{N,\tau e}^E \bar{\xi}_{N,\tau\mu}^E|^2 (F_1(y_{h_0}) - F_1(y_{A_0}))^2 + |\chi|^2 \\ &+ 2 |\bar{\xi}_{N,\tau e}^E \bar{\xi}_{N,\tau\mu}^E| (F_1(y_{h_0}) - F_1(y_{A_0})) |\chi| \cos \theta_\chi \cos (\theta_{\tau e} + \theta_{\tau\mu}) \end{aligned} \quad (10)$$

with  $\chi = e^{i\theta_\chi} |\chi|$ ,  $\bar{\xi}_{N,\tau e}^E = e^{i\theta_{\tau e}} |\bar{\xi}_{N,\tau e}^E|$  and  $\bar{\xi}_{N,\tau\mu}^E = e^{i\theta_{\tau\mu}} |\bar{\xi}_{N,\tau\mu}^E|$ . Of course, the amount of  $A_{CP}$  produced depends on the amount of new quantity introduced, however even a small contribution may bring a measurable  $A_{CP}$  for this process.

Now, we would like to discuss the similar analysis for another LFV decay,  $\tau \rightarrow \mu\gamma$ . The decay width for this process is calculated by taking only the  $\tau$ -lepton as an internal one [14] and it reads as

$$\Gamma(\tau \rightarrow \mu\gamma) = c_2(|B_1|^2 + |B_2|^2), \quad (11)$$

where

$$\begin{aligned} B_1 &= Q_\tau \frac{1}{48 m_\mu m_\tau} \bar{\xi}_{N,\tau\mu}^E \left\{ \bar{\xi}_{N,\tau\tau}^{E*} (G_1(y_{h_0}) + G_1(y_{A_0})) + 6 \bar{\xi}_{N,\tau\tau}^E (F_1(y_{h_0}) - F_1(y_{A_0})) \right\}, \\ B_2 &= Q_\tau \frac{1}{48 m_\mu m_\tau} \bar{\xi}_{N,\mu\tau}^{E*} \left\{ \bar{\xi}_{N,\tau\tau}^E (G_1(y_{h_0}) + G_1(y_{A_0})) + 6 \bar{\xi}_{N,\tau\tau}^{E*} (F_1(y_{h_0}) - F_1(y_{A_0})) \right\}, \end{aligned} \quad (12)$$

and  $c_2 = \frac{G_F^2 \alpha_{em} m_\tau^5}{32\pi^4}$ . Here the amplitudes  $B_1$  and  $B_2$  have right and left chirality, respectively. The function  $F_1(w)$  is given in eq. (7) and  $G_1(w)$  is

$$G_1(w) = \frac{w(2 + 3w - 6w^2 + w^3 + 6w \ln w)}{(-1 + w)^4}.$$

$A_{CP}$  in this process vanishes in the model III, similar to the one in the decay  $\mu \rightarrow e\gamma$  and we will follow the same procedure given above. By correcting the combination  $\bar{\xi}_{N,\tau\mu}^E (\bar{\xi}_{N,\tau\tau}^{E*} (G_1(y_{h_0}) + G_1(y_{A_0})) + 6 \bar{\xi}_{N,\tau\tau}^E (F_1(y_{h_0}) - F_1(y_{A_0})))$  with the additional small and complex quantity  $\rho$  due to the physics beyond the model III as

$$\bar{\xi}_{N,\tau\mu}^E (\bar{\xi}_{N,\tau\tau}^{E*} (G_1(y_{h_0}) + G_1(y_{A_0})) + 6 \bar{\xi}_{N,\tau\tau}^E (F_1(y_{h_0}) - F_1(y_{A_0}))) + \rho,$$

we get

$$A_{CP} = \frac{\Lambda}{\Omega} \quad (13)$$

where

$$\begin{aligned} \Lambda &= 2 |\bar{\xi}_{N,\tau\mu}^E \bar{\xi}_{N,\tau\tau}^{E*}| |\rho| \sin \theta_\rho \{ \sin(\theta_{\tau\mu} - \theta_{\tau\tau}) (G_1(y_{h_0}) + G_1(y_{A_0})) \\ &\quad + 6 \sin(\theta_{\tau\mu} + \theta_{\tau\tau}) (F_1(y_{h_0}) - F_1(y_{A_0})) \}, \end{aligned} \quad (14)$$

and

$$\begin{aligned} \Omega &= |\bar{\xi}_{N,\tau\mu}^E \bar{\xi}_{N,\tau\tau}^{E*}|^2 \left( (G_1(y_{h_0}) + G_1(y_{A_0}))^2 + 36 (F_1(y_{h_0}) - F_1(y_{A_0}))^2 \right) + |\rho|^2 \\ &\quad + 12 |\bar{\xi}_{N,\tau\mu}^E \bar{\xi}_{N,\tau\tau}^{E*}|^2 (F_1(y_{h_0}) - F_1(y_{A_0})) (G_1(y_{h_0}) + G_1(y_{A_0})) \cos 2\theta_{\tau\tau} \\ &\quad + 2 |\bar{\xi}_{N,\tau\mu}^E \bar{\xi}_{N,\tau\tau}^{E*}| |\rho| (G_1(y_{h_0}) + G_1(y_{A_0})) \cos \theta_\rho \cos(\theta_{\tau\mu} - \theta_{\tau\tau}) \\ &\quad + 12 |\bar{\xi}_{N,\tau\mu}^E \bar{\xi}_{N,\tau\tau}^{E*}| |\rho| (F_1(y_{h_0}) + F_1(y_{A_0})) \cos \theta_\rho \cos(\theta_{\tau\mu} + \theta_{\tau\tau}), \end{aligned} \quad (15)$$

with  $\rho = e^{i\theta_\rho} |\rho|$  and  $\bar{\xi}_{N,\tau\tau}^E = e^{i\theta_{\tau\tau}} |\bar{\xi}_{N,\tau\tau}^E|$ . Similar to the process  $\mu \rightarrow e\gamma$ , the amount of  $A_{CP}$  strongly depends on the new quantity introduced and a small contribution may bring measurable  $A_{CP}$  for this process also.

### 3 Discussion

In the case of vanishing charged interactions, with the assumption that CKM type matrix in the leptonic sector does not exist, LFV interactions are possible in the one-loop, due to neutral Higgs bosons  $h^0$  and  $A^0$ , in the framework of model III. In general, the Yukawa couplings  $\bar{\xi}_{N,ij}^E, i, j = e, \mu, \tau$  appearing in the expressions are complex and they ensure non-zero lepton EDM. However, this scenario is not enough to get a CP violating asymmetry in the LFV processes,  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu\gamma$ . To obtain such asymmetry, we introduce a small model independent correction term to the decay width with the following features:

- this term is due to the physics beyond the model III
- it is complex and the complexity does not come from the Yukawa type couplings

The additional contributions respecting the above conditions bring non-zero  $A_{CP}$  to both processes underconsideration. However, this extra quantity is completely unknown and the number of parameters, namely complex Yukawa couplings and new model independent quantity, increases in the numerical calculations. To solve this problem, first, we assume that the Yukawa couplings  $\bar{\xi}_{N,ij}^E, i, j = e, \mu$ , are small compared to  $\bar{\xi}_{N,\tau i}^E, i = e, \mu, \tau$  since the strength of them are related with the masses of leptons denoted by the indices of them, similar to the Cheng-Sher scenario [15]. Further, we take  $\bar{\xi}_{N,ij}^E$  symmetric with respect to the indices  $i$  and  $j$ . Therefore only the combination  $\bar{\xi}_{N,\tau\mu}^E \bar{\xi}_{N,\tau e}^E$  (the couplings  $\bar{\xi}_{N,\tau\tau}^E$  and  $\bar{\xi}_{N,\tau\mu}^E$ ) for the process  $\mu \rightarrow e\gamma$  ( $\tau \rightarrow \mu\gamma$ ) plays the main role in our analysis.  $\bar{\xi}_{N,\tau\mu}^E \bar{\xi}_{N,\tau e}^E$  can be restricted using the experimental upper limit of the  $BR$  of the process  $\mu \rightarrow e\gamma$  [1],

$$BR(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}. \quad (16)$$

(see [14] for details). Here, we do not take the contribution of additional part coming from the physics beyond model III since we assume that its effect on the constraint region is sufficiently small. Note that we take the additional contribution  $|\chi|$  as two orders smaller compared to  $\bar{\xi}_{N,\tau e}^E \bar{\xi}_{N,\tau\mu}^E (F_1(y_{h_0}) - F_1(y_{A_0}))$ .

For the process  $\tau \rightarrow \mu\gamma$ , the coupling  $\bar{\xi}_{N,\tau\mu}^E$  is restricted using the experimental upper and lower limits of  $\mu$ -lepton EDM ([16])

$$0.3 \times 10^{-19} e - cm < d_\mu < 7.1 \times 10^{-19} e - cm \quad (17)$$

(see [14] for details) and we do not use any constraint for the coupling  $\bar{\xi}_{N,\tau\tau}^E$ . For this decay, the additional contribution  $|\rho|$  is taken as two orders smaller compared to  $\bar{\xi}_{N,\tau\mu}^E (\bar{\xi}_{N,\tau\tau}^{E*} (G_1(y_{h_0}) + G_1(y_{A_0})) + 6 \bar{\xi}_{N,\tau\tau}^E (F_1(y_{h_0}) - F_1(y_{A_0})))$ , similar to the previous process.

Fig. 2 represents the new quantity  $|\chi|$  dependence of  $A_{CP}$  for  $\sin\theta_{\tau\mu} = 0.5$ ,  $m_{h^0} = 85 \text{ GeV}$  and  $m_{A^0} = 95 \text{ GeV}$ . Here the solid lines show the case where  $\sin\theta_\chi = 0.1$  and they correspond to  $\sin\theta_{\tau e}$  values 0.1, 0.5 and 0.8 in the order from the lower one to the upper. Similarly the dashed (small dashed) lines represents the case where  $\sin\theta_\chi = 0.5$  ( $\sin\theta_\chi = 0.8$ ). Choosing  $|\chi|$  at the order of  $10^{-7}(\text{GeV}^2)$ ,  $A_{CP}$  for the process can be at the order of the magnitude  $10^{-2}$ . As shown in this figure,  $A_{CP}$  is sensitive to the CP parameters  $\sin\theta_{\tau\mu}$ ,  $\sin\theta_{\tau e}$  and obviously to  $\sin\theta_\chi$ .

In Fig. 3 we present  $\sin\theta_\chi$  dependence of  $A_{CP}$  for  $|\chi| = 10^{-7}(\text{GeV}^2)$ ,  $\sin\theta_{\tau e} = 0.5$ ,  $m_{h^0} = 85 \text{ GeV}$  and  $m_{A^0} = 95 \text{ GeV}$ . Here, the solid line corresponds to  $\sin\theta_{\tau\mu} = 0.1$ , dashed line to  $\sin\theta_{\tau\mu} = 0.5$  and small dashed line to  $\sin\theta_{\tau\mu} = 0.8$ .  $A_{CP}$  increases with increasing values of the parameters  $\sin\theta_\chi$ ,  $\sin\theta_{\tau e}$  and  $\sin\theta_{\tau\mu}$ .

Now we would like to show our results for the  $A_{CP}$  of the process  $\tau \rightarrow \mu\gamma$  in series of Figures 4 -8. Fig. 4 is devoted to the new quantity  $|\rho|$  dependence of  $A_{CP}$  for  $\bar{\xi}_{N,\tau\tau}^E = 100 \text{ GeV}$ ,  $\sin\theta_\rho = 0.1$ ,  $\sin\theta_{\tau\mu} = 0.5$ ,  $m_{h^0} = 85 \text{ GeV}$  and  $m_{A^0} = 95 \text{ GeV}$ . Here,  $A_{CP}$  is restricted between solid lines for  $\sin\theta_{\tau\tau} = 0.1$ , dashed lines for  $\sin\theta_{\tau\tau} = 0.5$  and small dashed lines for  $\sin\theta_{\tau\tau} = 0.8$ . Note that the upper and lower bounds for  $A_{CP}$  is due to the constraint of  $\bar{\xi}_{N,\tau\mu}^E$  coming from the experimental limits of  $\mu$ -lepton EDM.  $A_{CP}$  for this process is at the order of the magnitude of  $10^{-3}$ . However, the increasing values of  $\sin\theta_\rho$  enhances it almost one order as seen in Figures 5 and 6, where they corresponds to  $\sin\theta_\rho$  values 0.5 and 0.8 respectively. The strong sensitivity of  $A_{CP}$  to the parameter  $\sin\theta_\rho$  is shown in Fig. 7. In this figure, we present  $\sin\theta_\rho$  dependence of  $A_{CP}$  for  $|\rho| = 0.1(\text{GeV}^2)$ ,  $\sin\theta_{\tau\mu} = 0.5$ ,  $m_{h^0} = 85 \text{ GeV}$  and  $m_{A^0} = 95 \text{ GeV}$ . Here,  $A_{CP}$  is restricted between solid lines for  $\sin\theta_{\tau\tau} = 0.1$ , dashed lines for  $\sin\theta_{\tau\tau} = 0.5$  and small dashed lines for  $\sin\theta_{\tau\tau} = 0.8$ . These figures show that the restriction region for  $A_{CP}$  becomes broader with increasing values of  $\sin\theta_{\tau\tau}$ . The same behaviour appears when  $\sin\theta_{\tau\mu}$  increases also.

Finally, we study the mass ratio  $m_{h^0}/m_{A^0}$  dependence of  $A_{CP}$  for the fixed values of  $\sin\theta_{\tau\mu} = \sin\theta_{\tau\tau} = 0.5$ ,  $|\rho| = 0.1(\text{GeV}^2)$  and  $m_{h^0} = 85 \text{ GeV}$  in Fig. 8. Here  $A_{CP}$  is restricted between solid lines for  $\sin\theta_\rho = 0.1$ , dashed lines for  $\sin\theta_\rho = 0.5$  and small dashed lines for  $\sin\theta_\rho = 0.8$ . It is observed that  $A_{CP}$  increases when the masses of neutral Higgs bosons are near to degeneracy. This enhancement is large for large values of  $\sin\theta_\rho$ .

As a result, it is possible to get a measurable  $A_{CP}$  for the LFV interactions  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu\gamma$  if there exists an additional small and complex contribution coming from the physics beyond the model III. Here, the complexity of the new contribution should not be due to the Yukawa type couplings, but comes from radiative corrections. With the reliable experimental

measurements of such asymmetries, it would be possible to test the new contributions and the corresponding physics.

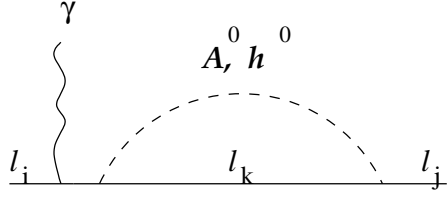
## 4 Acknowledgement

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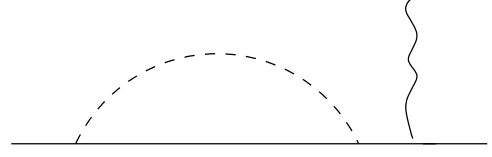
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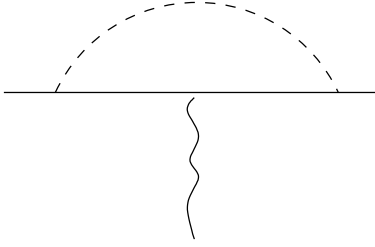




**( a )**



**( b )**



**( c )**

Figure 1: One loop diagrams contribute to the LFV decays  $l_i \rightarrow l_j \gamma$  with  $i \neq j$ . Solid line corresponds to the lepton, curly line to the electromagnetic field, dashed line to the fields  $h_0$  and  $A_0$ . Here  $l_k$  denotes the leptons  $e, \mu, \tau$ .

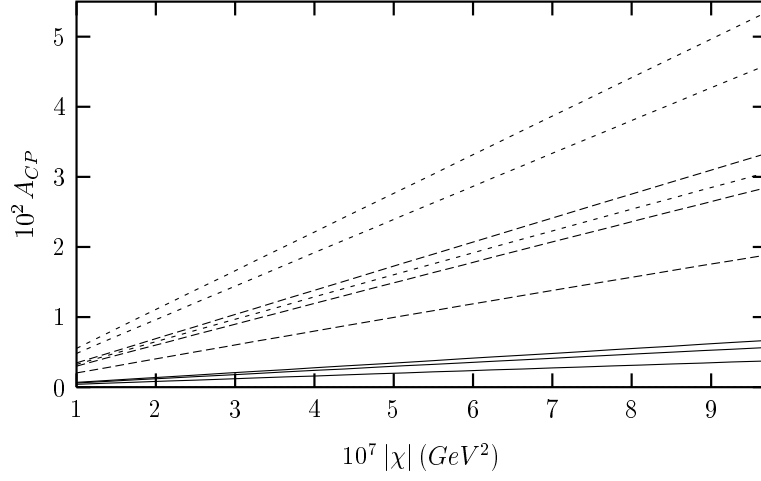


Figure 2:  $A_{CP}$  of the process  $\mu \rightarrow e\gamma$  as a function of  $|\chi|$  for  $\sin\theta_{\tau\mu} = 0.5$ ,  $m_{h^0} = 85 \text{ GeV}$  and  $m_{A^0} = 95 \text{ GeV}$ . Here, the solid lines show the case where  $\sin\theta_\chi = 0.1$  and they correspond to  $\sin\theta_{\tau e}$  values 0.1, 0.5 and 0.8 in the order from the lower one to the upper. Similarly, the dashed (small dashed) lines represent the case where  $\sin\theta_\chi = 0.5$  ( $\sin\theta_\chi = 0.8$ ).

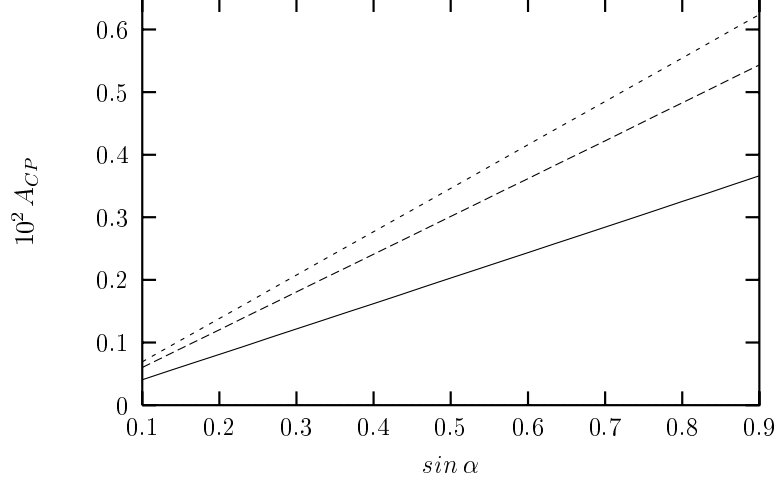


Figure 3:  $A_{CP}$  of the process  $\mu \rightarrow e\gamma$  as a function of  $\sin \theta_\chi$  for  $|\chi| = 10^{-7}(\text{GeV}^2)$ ,  $\sin \theta_{\tau e} = 0.5$ ,  $m_{h^0} = 85 \text{ GeV}$  and  $m_{A^0} = 95 \text{ GeV}$ . Here, the solid line corresponds to  $\sin \theta_{\tau\mu} = 0.1$ , dashed line to  $\sin \theta_{\tau\mu} = 0.5$  and small dashed line to  $\sin \theta_{\tau\mu} = 0.8$ .

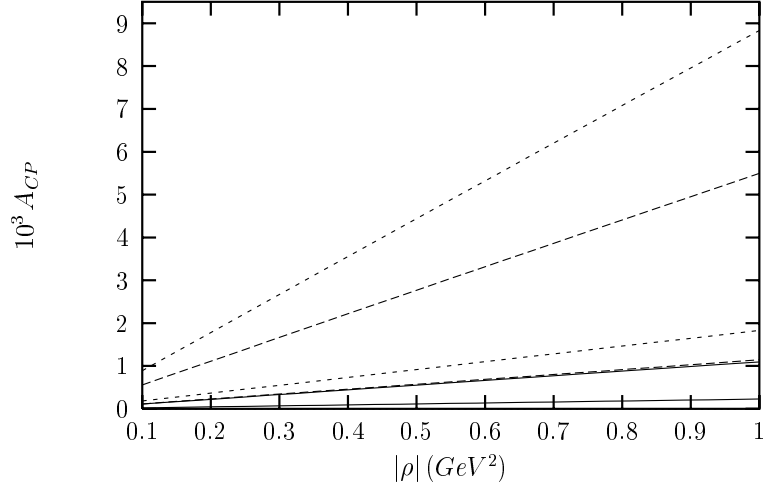


Figure 4:  $A_{CP}$  of the process  $\tau \rightarrow \mu\gamma$  as a function of  $|\rho|$  for  $\bar{\xi}_{N,\tau\tau}^E = 100 \text{ GeV}$ ,  $\sin \theta_\rho = 0.1$ ,  $\sin \theta_{\tau\mu} = 0.5$ ,  $m_{h^0} = 85 \text{ GeV}$  and  $m_{A^0} = 95 \text{ GeV}$ . Here,  $A_{CP}$  is restricted between solid lines for  $\sin \theta_{\tau\tau} = 0.1$ , dashed lines for  $\sin \theta_{\tau\tau} = 0.5$  and small dashed lines for  $\sin \theta_{\tau\tau} = 0.8$ .

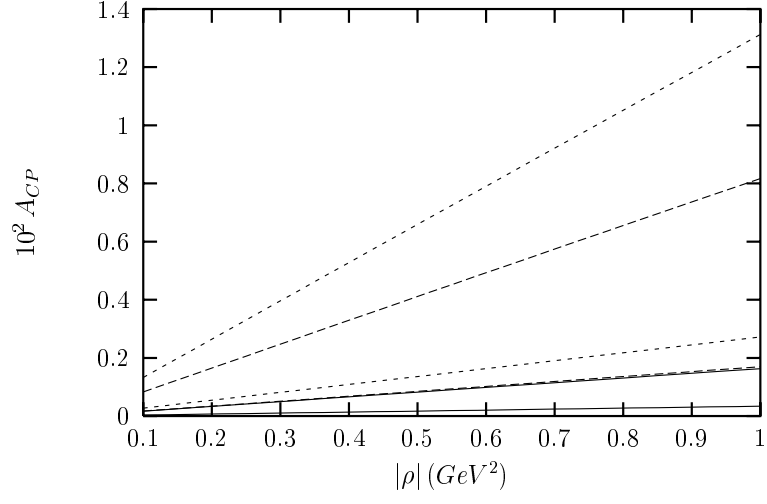


Figure 5: The same as Fig. 4 but for  $\sin\theta_\rho = 0.5$ .

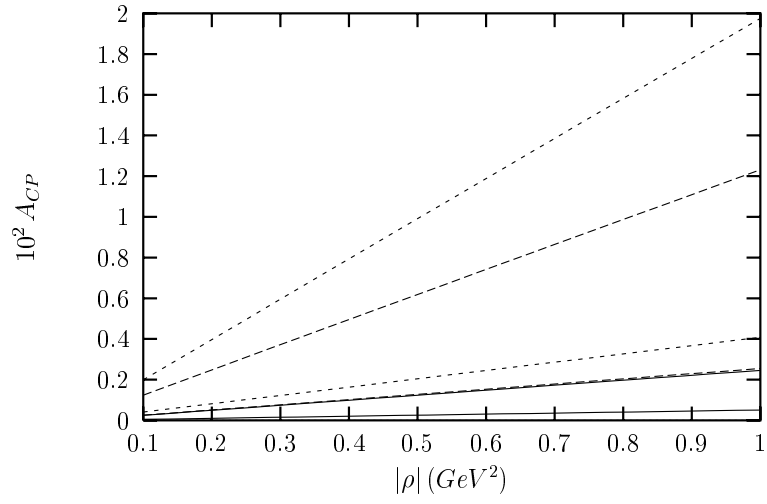


Figure 6: The same as Fig. 4 but for  $\sin\theta_\rho = 0.8$ .

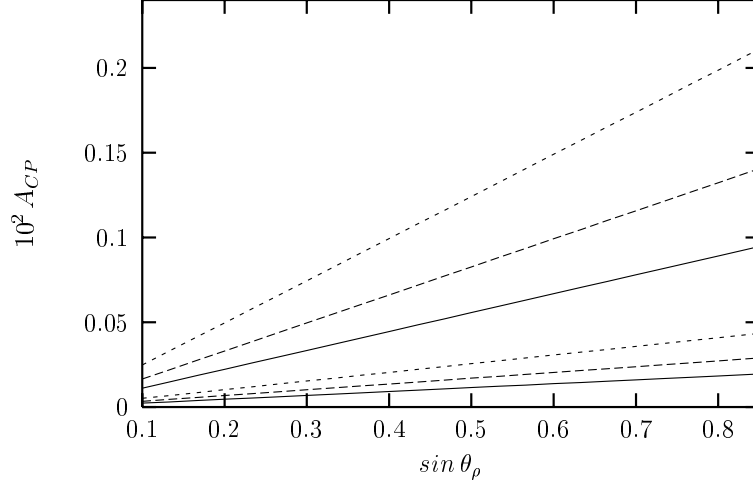


Figure 7:  $A_{CP}$  of the process  $\tau \rightarrow \mu\gamma$  as a function of  $\sin \theta_\rho$  for  $|\rho| = 0.1 \text{ (GeV}^2\text{)}$ ,  $\sin \theta_{\tau\mu} = 0.5$ ,  $m_{h^0} = 85 \text{ GeV}$  and  $m_{A^0} = 95 \text{ GeV}$ . Here,  $A_{CP}$  is restricted between solid lines for  $\sin \theta_{\tau\tau} = 0.1$ , dashed lines for  $\sin \theta_{\tau\tau} = 0.5$  and small dashed lines for  $\sin \theta_{\tau\tau} = 0.8$ .

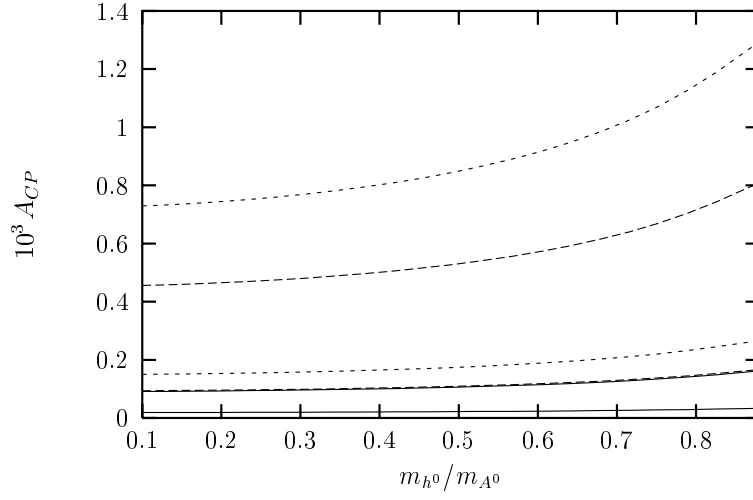


Figure 8:  $A_{CP}$  of the process  $\tau \rightarrow \mu\gamma$  as a function of the mass ratio  $m_{h^0}/m_{A^0}$  for  $\bar{\xi}_{\tau\tau}^E = 100 \text{ GeV}$ ,  $\sin \theta_{\tau\mu} = \sin \theta_{\tau\tau} = 0.5$ ,  $|\rho| = 0.1 \text{ (GeV}^2\text{)}$  and  $m_{h^0} = 85 \text{ GeV}$ .